Extending the fixed-points technique for optimum design of rotational inertial tuned mass dampers

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ABSTRACT

The fixed-points technique is an approximate version of H-infinity optimization and one of the most common methods used for the design of tuned vibration absorbers. The fixed-points technique is based on the existence of fixed points on the system's frequency response curve that are independent of the system's damping level and are thus at the same location during both the zero and infinite damping conditions. Optimum tuning parameter of tuned mass dampers (TMDs) have been obtained by equalizing the magnitude of the response at these fixed-points. This technique has been previously investigated for the optimal design of conventional TMDs and extended for various types of other TMDs. Recently, by replacing the damper in the TMD with combination of a tuning spring, viscous damper, and a small physical mass connected to a mechanism which converts translational motion to the rotational motion of that small mass, rotational tuned mass dampers (RITMDs) have been developed. However, the fixed-points technique has not been extended previously for RITMDs, which have one additional degree-of-freedom compared to TMDs. In this paper, the fixed-points technique is extended, via algebraic solution, for selecting the optimum tuning and damping values of RITMDs. Comparison of the response of the system with the optimum design values determined from the proposed method and numerical results in the literature demonstrates the validity of the assumptions and procedures of the proposed optimization method. Additionally, the performance of the system, in comparison to a conventionally optimized TMD via the fixed-points technique, shows that the RITMDs can be more effective at reducing the underlying system's maximum displacement response.
Keywords: Vibration absorber; Tuned mass damper; Fixed-points method; Optimum design; Rotational inertia damper

Introduction:

Fixed-points techniques have been proposed and utilized for finding the optimum tuning frequency and damping ratio of tuned mass dampers (TMDs) [1] (Figure 1) as an approximate version of H-infinity optimization. In addition, the fixed–points method has been used for optimum design of damped non-traditional TMDs [2], developed for TMDs attached to multi-degree of freedom primary systems [3], and generalized for global vibration control [4].

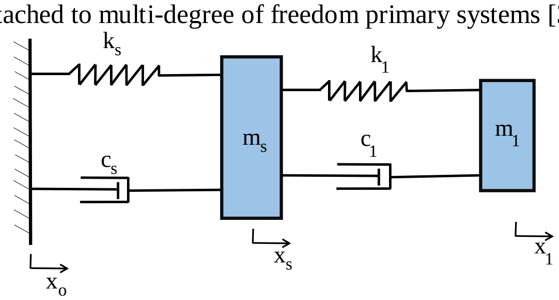

Figure 1: TMD

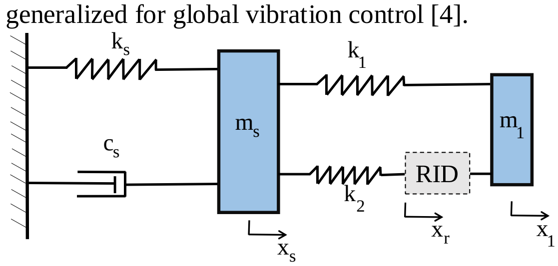

Figure 2: RITMD

Utilizing the inerter [5], which produces inertial mass through transferring the translational motion of the primary structure to the rotational motion of the inerter, different configurations of rotational mass dampers have been proposed and developed to control the response of the primary structure they are attached to and reduce the physical mass of the absorber [6]–[9]. As inerter based devices, rotational inertia tuned mass dampers (RITMD) (Figure 2), which consist of a TMD modified with the addition of a tuned of rotational inertia device (RID) such as rack and pinion or ball and screw mechanism, have been introduced and optimized numerically [10]. However, the fixed-point technique has not been developed for optimum design of RITMD. In this paper, a fixed-points technique is developed for optimum design of a RTIMD attached to an undamped SDOF primary structure.

Fixed-Point Technique for RITMD

For traditional TMDs, there are always two fixed-points (P,Q) in the frequency response curve for case of damping equal to zero or infinity [1] (Figure 3). In other words, these points are independent from the damping level of the absorber (c), therefore, the primary structure frequency response curves always passed through these two points. The fixed-points can be found by setting the zero and infinity damping transfer functions equal to each other $\left(\|H\|_{c=0}=\|H\|_{c=\infty}\right)$. The optimum frequency ratio can be obtained by setting the magnitude of the two fixed points equal. This provides an optimization condition in which the curve has a peak at these points [1]. By putting the partial derivative of frequency response function respect to damping equal to zero at optimum frequency points, two optimum damping are achieved and the final optimum damping is the average of two damping values.

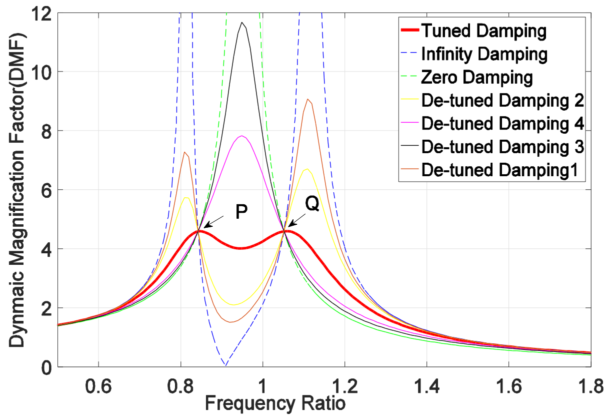

Figure 3 : Fixed-Points in the TMD's Frequency Response Curves

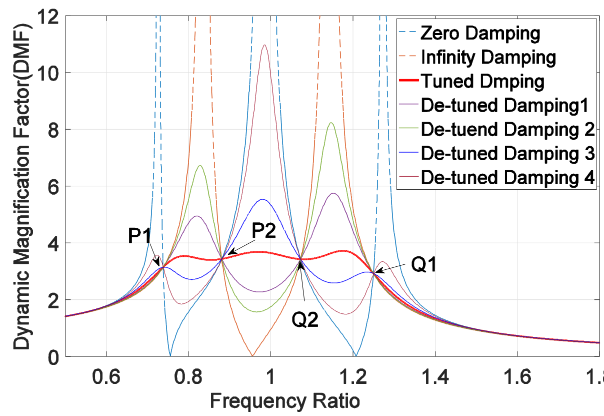

Figure 4: Fixed-Points in the RITMD's Frequency Response Curves

Since the RITMD is a three degree-of-freedom system with two optimum frequency ratios, four fixed-points (P_1,P_2,Q_1,Q_2) in the frequency response curves of the primary structure are observed (Figure 4). This observation can be support mathematically by solving the equation of zero-infinity damping in the RITMD $\left(\|H\|_{c=0}=\|H\|_{c=\infty}\right)$, which is a forth degree polynomial with four real positive roots. Extending the fixed-points method, we assumed the optimum frequencies condition occurs in the case of a pair equality of the primary structure response magnitude in the fixed-points $\left(\|H_{P1}\|=\|H_{P2}\|;\|H_{Q1}\|=\|H_{Q2}\|\right)$. This assumption leads to two high-order nonlinear equations which are solved numerically to find the optimum frequency ratios. In the final step, the optimum damping can be found by putting the maximum response magnitude of one of two frequency ratios equal to response magnitude of the other optimum frequency ratio.

Results and discussion:

To examine the proposed extended fixed-points technique accuracy, the frequency response of the primary system with optimum design values from the proposed method is compare with the optimum H-infinity design utilizing an numerical nonlinear programming optimization method [11]. The propose method is an approximate method, thus there are small difference in comparison to the exact H-infinity optimization utilizing nonlinear programming; however, the results from the proposed method are close, which demonstrates the accuracy of the assumptions in the extended proposed method. In addition, the response of the primary structure of both RITMD and TMD systems with the same secondary mass ratio (10%) and optimized with the fixed point method is shown in Figure 6. It can be observe that the RITMD exhibits superior performance in reducing the vibration amplitude of the primary system in compare to the TMD.

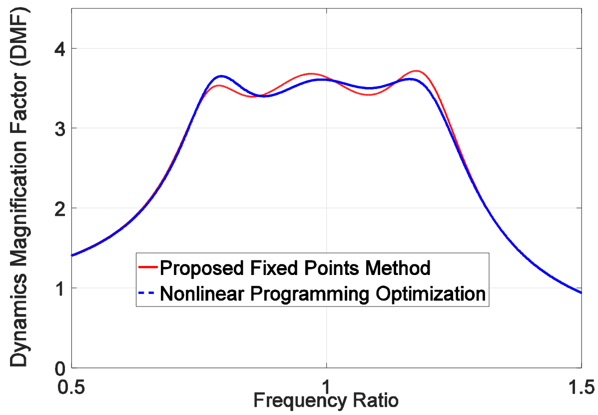

Figure 5: Extended Fixed-Point Method Vs Numerical Optimization; m1=10% ms (rotational mass=15% m1)

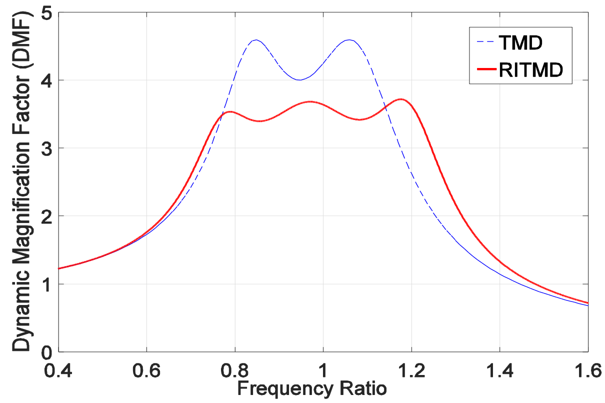

Figure 6: TMD vs RITMD Response; m1=10% ms (rotational mass=15% m1)

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